**Example**

**Data model**

\[ p(y|x) = \frac{1}{2} \mathcal{N}(y; x, 1) + \frac{1}{2} \mathcal{N}(y; 0, 10) \]

**Typical data**

**ADF posterior for three orderings of same data:**

True \( x = 2 \)

20 data points

ADF is sensitive to ordering

Can we make ADF independent of ordering?
Data model

\[ p(y|x) = \frac{1}{2} \mathcal{N}(y; x, 1) + \frac{1}{2} \mathcal{N}(y; 0, 10) \]

Typical data

EP posterior at convergence

Other methods

All independent of data ordering
Performance

Data size n=20

ADF = first ‘x’ of EP
VB = variational bound

Deterministic methods improve with more data
(posterior is more Gaussian)
Sampling methods do not care
Bayes point machine

Bayesian approach to linear classification

Use \( w \) to classify \( x \):

\[
\begin{align*}
    w^T x_i & > 0 \quad \text{(class 1)} \\
    w^T x_i & < 0 \quad \text{(class 2)}
\end{align*}
\]

\[
p(w, D) = p(w) \prod_i p(y_i | x_i, w)
\]

\( p(w) \) is uniform

\[
p(y|x, w) = \Theta(yw^T x) = \begin{cases} 
1 & \text{if } w \text{ is a perfect separator} \\
0 & \text{otherwise}
\end{cases}
\]

Classify a new data point by voting:

\[
p(y|x, D) = \int w \ p(y|x, w) p(w|D) dw
\]

\[
y = E[\text{sign}(w^T x)|D] \approx \text{sign}(E[w|D]^T x)
\]

\( E[w|D] \) is the Bayes Point
Bayes point machine example

SVM → Maximize margin
(distance to closest data point)

Bayes → Vote all perfect separators
Performance of EP

Billiard = Monte Carlo

Opper&Winther’s algs:
MF = mean–field theory
TAP = cavity method
(equiv to Gaussian EP)
Gaussian kernels

Map data into high dimensional space so that

\[ \phi(x_i) \T \phi(x_j) = \exp\left(-\frac{||x_i - x_j||^2}{2\sigma^2}\right) \]

SVM boundaries are more contrived, sensitive to kernel
Kernel selection

SVM and EP have similar boundaries, but prefer different kernels

<table>
<thead>
<tr>
<th>Kernel</th>
<th>$R^2/\rho^2$</th>
<th>log($p(D)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.08$</td>
<td>18</td>
<td>-39</td>
</tr>
<tr>
<td>$\sigma = 0.6$</td>
<td>108</td>
<td>-19</td>
</tr>
<tr>
<td>quadratic</td>
<td>656</td>
<td>-16</td>
</tr>
</tbody>
</table>